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NOTIONS OF PRACTICAL GEOMETRY OF VASCO DE ARAUJO E SILVA

NOÇÕES DE GEOMETRIA PRÁTICA DE VASCO DE ARAUJO E SILVA

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ABSTRACT

The text deals with the work Basic Ideas of Practical Geometry for the use of elementary schools published in 1869, in Porto Alegre, Brazil. Through a documental analysis, he contextualizes the place of production, identifies author's biographical data and seeks to answer the research question: what is the conception of teaching and learning elementary geometry that can be identified in the work Notions of Practical Geometry for use by schools of elementary instruction of Vasco de Araujo e Silva? For the analysis of contents presented in the text, he uses the following categories: figures, theoretical statements, exercises and proposed problems. We conclude that the geometric knowledge for primary education should be practical, elementary, that the teaching should begin with the freehand drawing and manipulation of geometric instruments. Learning geometry, for the author, implies visualizing, constructing geometric objects through step-by-step instructions, as well as exercising in a gradual manner without much theorizing.

Keywords: Primary Education. Geometry. Vasco de Araujo e Silva. XIX Century.

RESUMO

O texto trata da obra Noções de geometria pratica para uso das escolas de instrucção elementar, publicada em 1869, em Porto Alegre de autoria de Vasco de Araujo e Silva. Por meio de uma análise documental, contextualiza o lugar de produção, identifica dados biográficos do autor e procura responder a questão investigativa: qual a concepção de ensinar e aprender geometria elementar que a obra Noções de geometria pratica para uso das escolas de instrucção elementar de Vasco de Araujo e Silva permite identificar? Para a análise de conteúdos apresentados no texto, utiliza as seguintes categorias: figuras, enunciados teóricos, exercícios e problemas propostos. Conclui que os saberes geométricos para o ensino primário deveriam ser práticos, elementares e que o ensino deveria começar pelo desenho a mão livre e manuseio de instrumentos geométricos. Aprender geometria, para o autor, implica em visualizar, construir objetos geométricos por meio de instruções passo a passo, exercitar-se de maneira gradual e sem muitas teorizações.

Palavras- Chave: *Ensino Primário. Geometria. Vasco de Araujo e Silva. Século XIX.*

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INTRODUCTION

> The present treatise on practical geometry, especially for primary schools ... It is yet another book that I have compiled for public instruction, certain that no one will be unaware of the great usefulness of studying practical geometry, prompt assistance to the entire industrial profession (SILVA, 1869, p. 3).

The above fragment, included in the preface of the book Notions of Practical Geometry, shows that it is an initiation into the study of elementary geometry for public primary education. Practical knowledge is included among the existing knowledge orders. As Burke (2016) says, it is those knowledge and skills that need to be learned in practice, such as craft knowledge and can often be obtained by trial and error. One can think of practical geometry as opposed to theoretical geometry. However, the meaning the author used in his book has yet to be unveiled. Arriada and Nogueira (2014), when researching the production of school texts in the elementary school in Rio Grande do Sul, cite the Practical Geometry of Vasco de Araujo e Silva, pointing out that this author is one of the most requested in the last decades of the 19th century, in Rio Grande do Sul, together with Hilario Ribeiro, Jose Teodoro de Souza Lobo, Bibiano de Almeida, Eudoro Berlink, Demétrio Ribeiro and João Frankenberg.

Vasco de Araujo e Silva's textbook, taken as one of the sources in the research presented here, aims to broadening the investigations already carried out on the History of Mathematical Education in Brazil, once agreeing with Valente (2008, p. 141) "the historical trajectory of constitution and development of school mathematics in Brazil can be read in textbooks".

Referring to historical research on textbooks, Choppin (2004) warns of the need to take into account the multiplicity of agents involved in the life of a school work, which begins with participants in production, circulation and includes conservation for future generations. In this sense, a historical analysis of textbooks is an extremely complex activity. It cannot be isolated, focused exclusively on the knowledge contained in it, but must be accompanied by other sources, such as journals, legislation, bibliographic dictionaries, reports of public education agents, pedagogical journals, among other documents that allow it can see the textbook inserted in a broader system (DASSIE & COSTA, 2014). In the present paper, I use the periodical A Federação, the Legislation on Public Instruction in Rio Grande do Sul, Pedagogical Magazines, Reports of Provincial Presidents, Bibliographic Dictionary, as well as recent research on the subject of School Geometry. Maria Célia Leme da Silva (2018) listed six textbooks from 19th-century, three of them were considered foreign adaptations or translations: Albuquerque, (1829); Portella, (1854); Calkins[1], (1950) and three were considered the first Brazilian produtions: Borges, (1876); Pacheco, (1881); Freire, (1894). She indicated the presence of two proposals for the elementary teaching of geometry, both linked to drawing.

In addition to the works indicated by Leme da Silva, a textbook of geometry for primary education, still little known on the national scene - the Vasco de Araujo e Silva textbook, appeared in the far south of the country. It is one of the rare textbooks dedicated to school geometry published in Rio Grande do Sul in the 19th century. The text I present here is intended to make known to the public who was the author of this textbook and seeks to answer the following investigative question: "What is the conception of teaching and learning elementary geometry that the work Notions of Practical Geometry for school use of Vasco de Araujo e Silva's elementary instruction allow you to identify?

1 In 1886, this book was translated into Portuguese by Rui Barbosa.

The scant appreciation and disregard of the textbook as a source of research by historians is partly due to the difficulty of access to collections and also "their incompleteness", as pointed out by Arriada and Nogueira (2018, p. 52). In the nineteenth century, only in large cities were books published, which made it difficult to access such prints. However, in the province of São Pedro do Rio Grande do Sul, an editorial market has flourished, not only in the capital, but in inland cities such as Pelotas and Rio Grande. Anne Marie Chartier (2018, p. 7) pointed out that, "From 1880 to 1890, when the province of São Pedro do Rio Grande do Sul almost had a monopoly to school production for state public schools, the 'regionalist' selection criterion was also 'ideological' ". Prior to this editorial boom, in 1869, the year of Vasco de Araujo e Silva's Edition of Practical Geometry far south of Brazil, with a theme that until then had received little attention from the textbook authors.

EDUCATIONAL CONTEXT OF THE PROVINCE OF SÃO PEDRO DO RIO GRANDE DO SUL

The Farroupilha Revolution (1835-1845) left an unpromising legacy to the Province of São Pedro do Rio Grande do Sul in different aspects, including formal education, which did not even have an organized educational structure (ARRIADA & NOGUEIRA, 2014). The next two decades were a period centered on the regulation of education, the establishment of rules for its implementation and inspection, the search for an efficient teaching method, investment in the training of primary teachers (since there was no such school), the organization of the teaching career, among other measures.

As education began to be organized, the government assumed more and more power over it, although private enterprise continued to operate with relative freedom. In this drive to control teaching in public schools, as Arriada and Nogueira (2014, p. 175) tell us, the authorities' practice was: "[...] to determine what should be taught and which texts would be allowed: in public schools may only be admitted to competently authorized books ", according to the Public Instruction Regulation, in 1869.

Porto Alegre was at the time a city with economic life centered on two poles: on one side, the river port and, on the other, the political and administrative functions linked to the condition of capital of the Province of São Pedro do Rio Grande do Sul (SILVA, 2016, p. 27).

In this context, a textbook author has emerged whose preface to the book Notions of Practical Geometry, says he believes his teaching was very useful and even more necessary for the industrial profession. This preoccupation with relating geometric design to industrialization is *somewhat* strange, since the Province at the time had a large land structure. According to Luvizotto (2009), it was only from 1875, with the arrival of Italian immigrants, an industrialization axis between the capital and Caxias do Sul was established.

The Jornal do Comercio Typography edited books in the provincial capital. In the nineteenth century, according to Arriada (2012), before the establishment of an editorial network, the printers performed this function, mainly to meet the needs of editing educational material for schools. The printers appeared not only in Porto Alegre, but also in inland cities.

TRACES OF AN AUTHOR FROM RIO GRANDE DO SUL IN THE 19TH CENTURY

For Ginzburg (2007), in the search for local stories or those little known characters, we need to search for trails, those almost invisible clues that allow us to slowly unveil, subject, objects or situations that we are trying to know.

The identification of Vasco de Araujo e Silva's date of birth in 1838 was made possible by a long search in the newspapers of the time (the website of the Digital Library of the National Library of Rio de Janeiro was used). The small note, of November 25, 1895, from the newspaper A Federação, records the death of our character on November 23, complementing with professional information: "He was a private teacher, was a state civil servant and collaborated on several pages of this capital. He was 57 years old. He left widow and minor children". In addition, it reports that he was white and the cause of death was Cachesia tuberculosa (REGISTRO MORTUÁRIO, 1895, p. 2).

Another clue was found in a brief biographical note in Blake's Brazilian Bibliographic Dictionary (1902), identifying him as the son of General Gabriel de Araujo e Silva and Mrs. Josephina Leopoldina da Silva Guimarães. He was born in Porto Alegre. He completed his studies at the Military and Application School of the Province of São Pedro do Rio Grande do Sul, and he obtained the rank of the first Cadet in February 1859 (Report of the Ministry of War 1859-1940). He went to study at the Central School of Rio de Janeiro. His name appears as passed both physics and math exams in November 1859 and chemistry in November 1860. We do not know if he completed his studies at this institution. Our hypothesis is that he did not conclude, since all the journalistic articles and reports found do not give him the title of bachelor, only citizen Vasco de Araujo e Silva, nor is there any reference to a title on the book's cover. However, passing the math, physics, and chemistry subjects at Central School provided some basis for writing the textbooks he edited.

The Presidents' Reports (available at the Porto Alegre Legislative Assembly Digital Library) read that in 1870, the Provincial Directorate ordered the purchase of one thousand copies of the Geography of Vasco de Araujo e Silva (RELATORIOS DOS PRESIDENTES, 1870, p. 7). The full title of the textbook appeared in 1885, when the editor of the newspaper reported receiving from the author a copy of his short book Notions of General Geography (UNTITLE, 1885, p. 1). In the same announcement, it claims to be the 3rd edition of the work, which had been used in public schools. Arriada states that the Rio-Grandense College, established in 1870, was created by the brothers Apolinario and Apeles Porto Alegre, together with Vasco de Araujo e Silva (ARRIADA, 2007, p. 105).

The involvement of Vasco de Araujo e Silva in the province's educational sphere has intensified since the 1870s. His appointment in 1871 to join as a fifth member of education director council (José Bernardino da Cunha Bittencourt, Archbishop Canon Zeferino Dias Lopes, Major Joao Luiz de Andrade Vasconcellos, Porfirio Barbosa Madureira) reveals his participation not only as a textbook author, but also as someone involved in the administration and definition of education in the Province. The circulation of the author's book on practical geometry can be confirmed in newspaper advertisements, for example, in 1872, the Arsenal de Guerra bought 60 copies (EDITAES, 1872, p. 3).

In 1892, a brief report informed that the adoption of Vasco de Araujo e Silva's Notions of Agriculture, which had the price of 300 R$, was authorized in the state's public schools. (ACTOS DO GOVERNICHO, 1892, p. 2). In 1897, in the Almanak Literary and Statistical (RS), there is a more

complete mention of the book's title, Notions of Agriculture for use by primary schools, by the same author, which was commissioned by the Government (GUIMARÃES, 1897, p. 6).

These traces allowed us to know at least three textbooks written by the author. Confirming what Arriada and Nogueira (2014) tell us about government control, we find the power exercised by the state, as an appraiser of educational works and as that person who authorizes or not the use of books in public schools. As a censor, he sanctions those textbooks that could be used in schools and even more, he buys the recommended works for distribution in public institutions. The author, for his part, takes advantage of the widely circulated newspapers and uses strategies to publicize his book, sending a copy to the journal editor, as can be seen in an announcement by the editor of The Federation in 1872.

Some clues about our character's professional performance appear in the newspapers and reports. He was a private school teacher, a member of the Board of the Public Education Inspectorate (AVISO, 1872, p. 1), an official of the Office of the Secretariat of Government (1885), a school inspector (ANNUNCIO, 1872, p. 1), examiner of preparatory general examinations of Portuguese (EXAMES PREPARATORIOS, 1887, p. 2) and also held the position of Deputy Director of the Directorate of Statistics, Government Body (GOVERNO DO ESTADO, 1892, p. 2). Vasco Araujo e Silva's interest also turned to literature, working in literary associations such as the Gremio Literário in 1876 (MALA DO SUL, 1876, p. 1), as President, and on Parthenon Litterario where he was a board member of directors and became vice president. In addition, he gave lectures there, including Family Mothers Education of the Family (PARTHENON LITTERARIO, 1873, p. 324). He worked in the writing of the Popular Diary in the early 1880s (Rio Grande do Sul, 1882, p. 2). This brief description of Vasco de Araujo e Silva's professional life allows us to conclude that he was not only a textbook author, but an active member of the local society and who excelled in public office. Moreover, by not including any bibliographic references in his geometry book and the lack of information about his formation, we can say nothing about possible influences of national or foreign authors.

A GEOMETRY TEXTBOOK FOR PRIMARY EDUCATION

The primary education legislation of 1827 already provided for, among other things, the inclusion of notions of practical geometry, according to Valente e Silva (2014). Two years later came the book by Holanda Cavalcanti, entitled Principles of Linear Design, including those of Practical Geometry, by the mutual teaching's method extracted from Francoeur. Although it is a compilation by a French author, it is one of the first works, which according to Valente e Silva (2014, p. 31) made it possible: "[...] to interpret the legislative demand for a practical geometry for teaching primary - shows that geometry is practical if the students are led to work with geometric figures. " Vasco de Araujo e Silva's Geometry book, as well as the others he wrote for teaching, contains in its title the word "notions", implying that this is an introduction to practical geometry.

Figure 1 - Book Cover

Source: PUC-RS Central Library Collection

In the list of definitions, presented at the beginning of the book, he explains what he means by speculative geometry and practical geometry, making it clear that he will not deal with "geometry science", which aims at the general and unchanging principles of figures, but which he will address that practical part, which is the art of solving the different issues of extension through graphical processes.

The subtitle is supplemented with important information - "mandated for adoption by the Public Education Board", which means that the book was used in schools. Making the cover of the book state that the work was recommended by the Council of Public Instruction meant giving it legitimacy. The government confirmed that his work could be used in public schools and this gave the text produced another status. A relevant document for the research in question is the Primary Instruction Regulation of April 5, 1869. In part, following the 1857 legislation which provided that primary schools would be of two grades: elementary education and 2nd grade - higher primary education, in this document there is a breakdown of what to teach in each of the grades. Specifically with regard to mathematical elementary knowledge, we have: for the 1st degree: "elementary principles of arithmetic and metrology", while for the 2nd degree: "arithmetic to proportions and their applications, practical geometry with application to arts and crafts " (SCHNEIDER, 1993, p. 253).

This local legislation corroborates that Silva and Valente (2013) found, in documents that regulate public instruction after Independence, the presence of geometry as a school knowledge to be worked on in primary schools. The notions of practical geometry integrated different programs.

Chapter 11 of the Rules explains the duration of the practical geometry course, as well as the distribution of assignments: "Every teacher should have for the study of practical geometry a good

ruler, a bar, a protractor (an instrument for measuring angles), a line drawer and a proportion scale " (SCHNEIDER, 1993, p. 265).

The indication of the books recommended for primary education appears in chapter 18,_and they are: *Arithmetica* by Diogo Francisco Cardoso and *Practical Geometry* by Vasco de Araujo e Silva. Thus, in the legislation, two "gauchos" authors were chosen to teach arithmetic and geometry. Referring to the practice of examinations, the Regulation is detailed and provides for oral examinations that the student, when examined in practical geometry, should describe in stone the figures on which he was accused.

A paratext that deserves discussion is the preface, because in it the author brings relevant clarifications to understand the proposal of the book. With the title of *To the reader*, the author basically brings four ideas: 1) to whom the book is intended; 2) the reasons for the division of the contents in the book; 3) utility of geometry; 4) solicitation of teachers' suggestions for text improvement. As already explained, the author wrote a book of geometry for beginners, for the "schools of primary education", in his words "within the reach of the less educated intelligences". He explains that the division used in the book is "natural" and has the advantage of arousing a taste for geometry without boring. We note that the author considered didactic aspects in the work, such as motivating the student and captivating him for the interest of building this knowledge. It should also be noted that what the author refers to as the natural division of geometry may correspond to beginning his studies by plane geometry, with lines and polygons, and then presenting spatial geometry, following the Euclidean tradition. He also says that practical geometry is important because it is an "aid to the entire industrial profession." Aware that his "compilation" has limitations, he requests: "I will ask the masters to advise me and I will improve it then in another edition " (SILVA, 1869, p. 3). So far, we have no indication that this book has been reprinted.

However, the authors Peres and Michel (2018) identified sources which prove that, in 1882, Vasco de Araujo Silva's Notions of Practical Geometry, as well as his General Geography book were in the list of textbooks in public schools in Porto Alegre.

GEOMETRICAL KNOWLEDGE

For the content analysis of the book *Notions of Practical Geometry*, there was initially no concern with framing it within any current pedagogical tendency, but unveiling the author's understanding of a method for teaching and learning geometry for a beginner. The strategy chosen was to elect some later categories that emerged from the text: figures, theoretical statements, exercises and proposed problems.

The 91-page book includes an appendix with figures (in addition to those in the appendix, the author numbered 106 figures in the text). It is divided into chapters, which the author calls "books," according to the Euclidean tradition: First Book, which comprises the notions of practical geometry; Book two: comprises the rectilinear figures; perpendicular lines; angles; triangles; quadrilaterals; polygons; division of the lines; division of angles; division of triangles; quadrangle division; Fourth book[2], which deals with mystilinous figures (basically problems of entering and circumscribing figures); Book Five: Application of the preceding notions to different problems; Book six: of the reasons and geometric proportions, scales, equivalent figures, volumes, measures of solids, the building solids way.

2 We do not find the reference of the third book, although there are no pages missing in the analyzed edition. I suppose there must have been an error enumerating the chapters, jumping from the second to the fourth.

He starts with a list of definitions. They are not totally formalized, as he sometimes adds an example, as in the definition of body: “Body is called anything that occupies a place in space, a table, a box, a book, etc. are bodies” (SILVA, 1869, p. 5). This articulation between the geometric objects and those of the student’s daily life points to an advance in Araujo e Silva’s proposal and distances itself from the more theoretical presentations of Euclidean geometry. For example, for Christiano Ottoni (1857, p. 1), author of textbooks contemporary to Vasco de Ara-ujo e Silva, the definition for body is more formalized: “The whole body occupies, in the infinite space that encompasses the universe, a definite or finite place, which is itself called a space”. There are similar definitions to those of Araujo e Silva in Abilio Cesar Borges’s book: “Surfaces have only two dimensions of extension, that is, length and width. Thus we say the surface of a cattail, a ball, a bottle, etc ” (BORGES, 1882, p. 2).

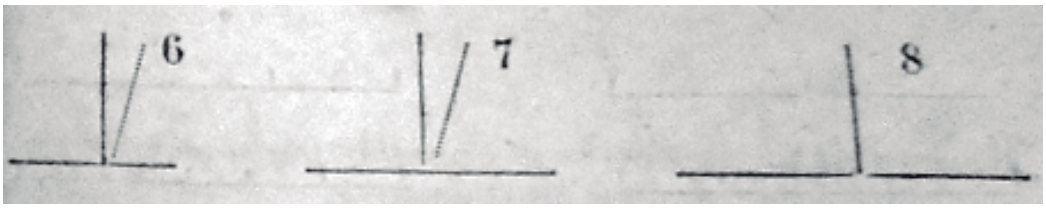
The comparison with the works of geometry by Ottoni and Borges is justified since they circulated in the province at the same time as that of Araujo e Silva. The announcement of the Livraria do Globo (ANNUNCIOS; 1890, ed. 46, p. 3), brings classic geometry books by French authors, with translations into Portuguese, such as: Legendre’s Geometry Elements; Lacroix’s with the same title; Briot’s book in French; as well as the national authors: Vasco de Araujo e Silva, Christiano Ottonio, Abílio Borges, Ayres Gama among others.

The first lesson in Vasco de Araujo Silva’s book, exemplarily, shows how the author believed that the beginner should proceed to acquire geometric knowledge. The author’s proposal to prepare and exercise the student in the most elementary notions of tracing geometric figures, starting with the line (currently called segments).

He suggests: “The student draws by hand on the slate more or less long straight lines and then corrects them with the ruler” (SILVA, 1869, p. 8). Following this text, he inserts three segment images and clarifies the procedures: “After he knows how to draw straight lines well, he tries to join, by a line, two points, first closer then more distant, always correcting with the ruler” (SILVA, 1869, p. 8). Following are two figures of one shorter and one longer segment. In the *Eschola Publica* magazine, almost three decades later, similar guidelines are read in Tolosa’s article entitled First Drawing Lessons, suggesting the use of dots to aid in line drawing (TOLOSA, 1893, p. 21).

The third proposed exercise is: “If the student is familiar with the two preceding exercises, he will draw a line perpendicular to the other, that is, a line that falls on another line does not lean either way” (SILVA , 1869, p. 9). In addition, it suggests that the student use the square to check if they have achieved the goal and exercise until they get a perfect job (Figure 2). According to Silva (2018), in the work of Albuquerque, translated and adapted by Francouer, there are freehand drawing practices in which hand skills will be used in tracing and eyes to make measurements.

Figure 2 - Plotting perpendicular lines.



Source: Silva, 1869, p. 9.

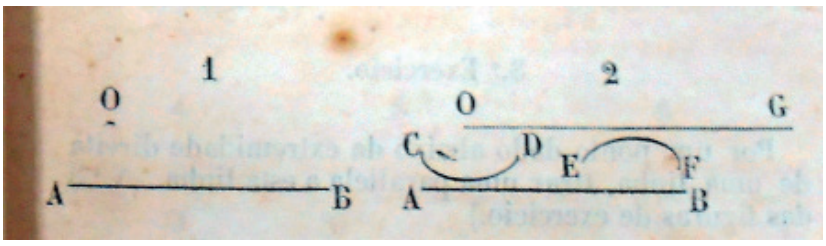
The method proposed by the author is trial and error - getting to know the geometric objects through freehand tracing, trying to build with the guidance of the teacher, who gives the characteristics of the objects. For example, to draw the line requires at least two points. According to Silva (2016), in Albuquerque's work (1829), the student will practice the imitation practice, which differs from our author's proposal. The geometric instruments such as ruler, compass and square are allowed, at this initial moment, only as instruments of verification of the constructions, because the goal is the free-hand tracing. It is worth highlighting the role that vision played in this practice, since the student would make estimates to be able to make the divisions of the segments without the use of instruments. The objective of the first part of the first author is this approximation of the freehand, straight, and circle drawing. Albuquerque's book presents only freehand straights.

THE FIGURES

The figures in Vasco de Araujo e Silva's book are of three types: 1) those of beginning - they are approximate geometrical figures, similar to those of a beginner, if he made them freehand, illustrated in Figure 2; 2) those built with geometric instruments; 3) those to illustrate definitions. At first I called the beginning figures because they seem to have the function of starting, by trial and error, the tracing of freehand figures, they are of the kind of those of Figure 2. The second are constructions with geometric instruments, they have the give more precision to the drawing and are closer to the traditional texts of geometric design, as they present step-by-step instructions on how to construct a figure. Example: 1st Problem - "By a point **O** given above an **AB** line, draw a parallel to this line" (SILVA, 1869, p. 15). The following is a figure composed of two parts, one where the parallel has not been drawn yet and the other one that has already been drawn. The solution is step- by- step given by the author, and needs the figure to help with construction procedures.

Set the compass point over point O, and describe a CD arc that just touches the line. Over the right end B, and with the same opening as the bar, another arc EF is described. From the given point you get a line OG, that just touches the last arc. This will be the parallel requested (SILVA, 1869, p.15).

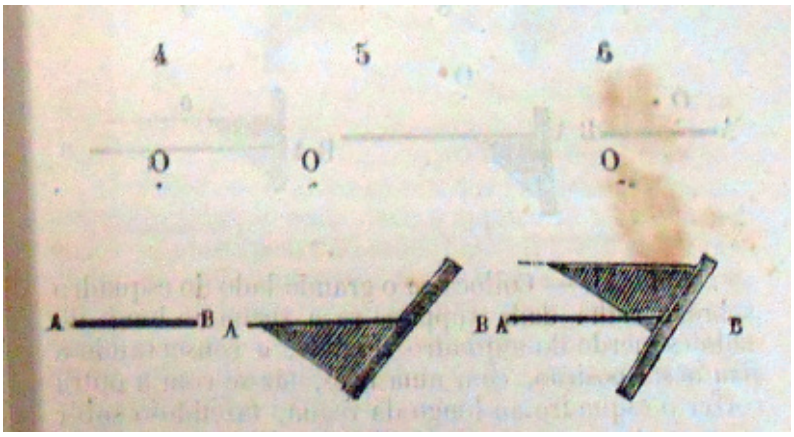
Figure 3 - Drawing of parallel line with compass aid.



Source: Silva, 1869, p. 15.

The figures in the appendix serve as a model for the student, according to what he/she makes in the footnote: "This exercise and the following should be done by the student, and then checked by using the exercise figures page" (SILVA, 1869, p. 15). The figures are essential to the text, because the descriptions follow step by step what should be done by the teacher and student, in the case exemplified in figure 4, should do the sequence of three images illustrates the doing:

Figure 4 - Parallel drawing with the help of the square.



Source: Silva, 1869, p. 17.

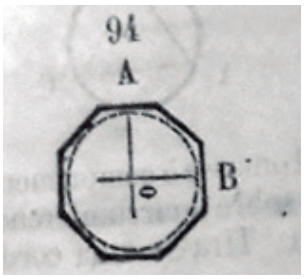
The proposed problem: “At a point given above the right end of a line draw a parallel to this line through the square” (SILVA, 1869, p. 17). As a solution, he explains step by step how the parallel line should be constructed. He proposes other similar problems on the same theme, changing the position of the point and the line, but following the same steps. This leads us to infer that these activities had the didactic purpose of exercising the student, as they are repetitive. As the text progresses, the author slightly changes the strategy of inserting the figures, leaving the image more summarized. This leads us to conjecture that the students, already exercised with the use of instruments (ruler and compass), would understand its construction without the need for intermediate steps. For example, to construct an equilateral triangle, it presents only one image (Figure 5).

Figure 5 - Construction of the equilateral triangle.

Source: Silva, 1869, p. 31.

For the problem of inscribing a circle in a regular octagon, the author proposes: “In the middle of any two sides rise perpendicular OA and OB. A center is made at the meeting point of the perpendiculars, and with a radius equal to one of them a circle is described ” (SILVA, 1869, p. 65). Figure 6 follows the problem.

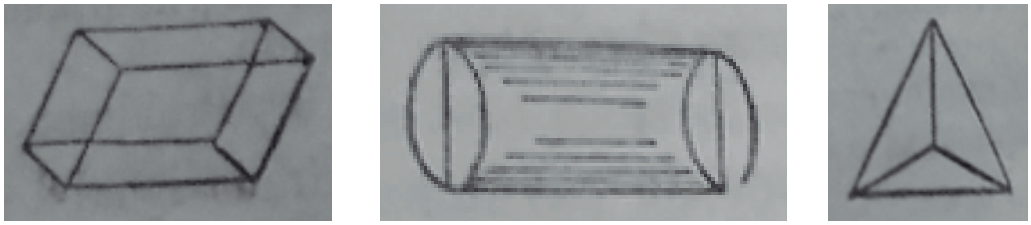
Figure 6 - Enclose in a regular octagon a circle.



Source: Silva, 1869, p. 64

A third type of figure appears to illustrate the definitions of spatial geometric beings, such as the prism, cylinder, and pyramid, some of which are exemplified in Figure 7. In this lesson, the author does not draw analogies between these geometric beings and everyday objects, as if the student no longer needed an embodiment to understand them.

Figure 7 - Geometric Solids.



Source: Silva, 1869, p. 84-85

THEORETICAL STATEMENTS

Theoretical statements mean the texts that precede the figures, and which resemble definitions, but were not so named by the author, although they fulfill this role. In the second book, we identify the following theoretical statements:

1. Parallel lines: “Parallel lines are those which, being drawn on the same plane, never meet, however long they extend” (SILVA, 1869, p. 14).

This formulation, which Araujo e Silva did not call a definition, resembles the definition of parallel lines in Euclid’s Elements: “Parallel are straights which, being on the same plane, and being extended unlimitedly on either side, nowhere meet” (EUCLIDES, 2009, p. 98).

2. Angles: “Angle is called the portion of the plane limited in part by two intersecting lines” (SILVA, 1869, p. 35).

This definition departs significantly from the Euclidean, which is understood as a slope rather than as a portion of the plane. However, Araújo’s (1999) historical research on the concept of angle in textbooks shows that a definition similar to Silva’s also appears in geometry textbooks by Brazilian authors such as Ottoni (1857): “Two lines AB, CD [fig. 3.2] that cut, divide the indefinite extent of the plane that they determine into four distinct portions, which are called angles. So, angle is called

the portion of a plane bounded in part by two intersecting lines. Silva appropriates part of Ottoni's definition and uses it without citing the author. In addition to defining angle, the author explains his understanding of right angle, acute and obtuse and also vertex.

3. Triangles: "A triangle is called the portion of a plane bounded by three straight lines". The elements of the triangle are also named and explained: "At any triangle, sides are called the lines that form it: base is either side, vertex is the vertex of the angle opposite the base, and height is the perpendicular lowered from the vertex over the base ". In addition, he defines triangle rectangle, acutangle, obtusangle, isosceles and equilateral. It is noteworthy that an important property of the triangle appears only as a footnote: "A triangle can have no more than one right angle, no more than one obtuse, one straight and the other obtuse" (SILVA, 1869, p. 30). This note is not accompanied by any justification why a triangle cannot have more than one right angle 4. Quadrilaterals: "A polygon is generally called any portion of the plane, completely closed or limited by straight lines" (SILVA, 1869, p. 34). It is interesting to note that this lesson is called "quadrangle", although he begins by defining polygon, then explains that triangle is the simplest polygon, and introduces other types of quadrangles and the concept of diagonal. In the text, the author sometimes formulates a problem statement and gives a footnote explanation. For example: "In a given circle ABCD form a perfect square." In a footnote he explains the meaning of the expression "to form": "It is called to inscribe a square in a circle" (SILVA, 1869, p. 64). After presenting various problems of inscribing figures in other given figures, he proposes problems of circumscribing figures without, however, providing any explanation of this term.In the fifth book, devoted to the applications of geometric notions: he begins with the definition of equal figures: "Equal figures are those which, applying one to the other, fit perfectly." This explanation resembles one of the common notions of Euclid's Elements: "And things which fit together are equal to each other" (EUCLIDES, 2009, p. 99).

The scales are an interesting topic addressed. The author relates scales to geographic charts, noting details that appear on those charts and explaining what scale is and what it is for, but does not provide a definition:

> Scale is a line that usually appears at the bottom of the geographic charts, divided into equal parts, each representing a mile, league, or any other distance, and serves to indicate the relationship in which each chart is to greatness of the earth (SILVA, 1869, p. 79).

Land measurement, another subject of practical geometry, aims to solve simple problems such as area calculation: "To measure a surface is to apply it as many times as possible, a certain determined and invariable surface, taken per unit of measure" (SILVA, 1869, p. 80).

Beginning with lesson 20, it starts spatial geometry, where you define volume, polyhedron, prism, parallelepiped, cylinder, pyramid, sphere, maximum circle of the sphere. It states that: "To evaluate or measure the volume of a body is to find the numerical relationship between this volume and another chosen for unity" (SILVA, 1869, p. 86).

EXERCISES AND PROBLEMS

Although it uses the terms 'problem' and 'exercise' in the text, the difference between the two is not very clear. Concepts can be introduced through both exercises and problems.

Table 1 - Exercise and Problem

Exercise	Problem
"Extend the perpendicular you have just drawn to form a cross from which each side fell perpendicular to the other two" (SILVA, 1869, p. 9).	"By a point O on a line AB (fig. 21) raise a perpendicular to this line through the square" (SILVA, 1869, p. 21).
"On a given line construct an isosceles triangle (fig. 18 - page of exercise figures)" (SILVA, 1869, p. 31).	"On a given line AB, build an equilateral triangle" (SILVA, 1869, p. 31).
"Plot a quadrilateral equal to a given quadrilateral (fig. 101)" (SILVA, 1869, p. 69).	"Plot a quadrilateral equal to a given quadrilateral (fig. 100)" (SILVA, 1869, p. 69).
"Finding the surface of a rhombus" (SILVA, 1869, p. 81).	"Measuring the surface of a 6 meter square" (SILVA, 1869, p. 80).

Source: data worked by the author

Comparing the statements in Table 1, it is not possible to understand the reason for using two different terms for tasks that are similar. That was the reason why we encompassed the terms exercise and problem in the same category. Among the suggested activities appear those of carton as: "Making, with cardboard, a regular triangular prism" (SILVA, 1869, p. 89); "Make a regular quadrangular prism"; "Make a cobblestone"; "Making a regular triangular pyramid" (SILVA, 1869, p. 90). The author does not present molds to building these models, but guides how to make the drawings and cut the figures. Comparing with Borges's book (1882, p. 85), we note that it also makes use of solid planning, calling it "development of solids surfaces", arguing that "[...] without such models it is almost impossible to the boys understanding the developments of solids ". In my opinion, this suggestion to use the carton box plays a didactic role in the text. It is a way of making learning more accessible to children in primary education. Problems are sometimes entered into "formulas" for resolution. For example: "Find the volume of any pyramid. Solution: Multiply the base by one third of the height" (SILVA, 1869, p. 87). Rare are the numerical examples like this:

> Example - Be a triangular pyramid whose base is a triangle that_has a height of 16 feet and 10 of base and whose height is 42 feet. Search for the surface of the base triangle by multiplying 10 by 8, half the height of the triangle, and the result 80 feet is multiplied by one third of the height of the pyramid. Therefore 80x14 = 1120 feet, volume requested.

There is no theoretical discussion of how to calculate the volume of solids, but rather a lightened presentation of the concept of volume by the presentation of the solving formula without mathematical symbology.

The author avoids the introduction of formulas, preferring to use natural language in resolutive description. One hypothesis for this omission may be that it is a book for primary education when pupils have not yet been introduced to algebraic concepts. Although Borges's (1882) presentation of spatial geometric beings is more broader than Silva's, we find no reference to the calculus of volumes, as we have exemplified earlier.

Practical geometry appears in the nineteenth-century legislation in the province of São Pedro do Rio Grande do Sul, not only as a piece of knowledge to integrate the education of primary school students, but also of military schools, competitions for public agencies, Normal School programs and preparatory exams. Until the 1930s, knowledge of practical geometry was required and appears

widely in periodical newspaper advertisements such as The Federation. For example, in 1905, for the public tender for state rural schools, Practical Geometry was the agenda of the program (PROGRAMMA PARA OS CONCURSOS DAS ESCOLAS RURAES DO ESTADO, 1905, p. 1); in the 1906 Public Instruction Regulation, for primary education the inclusion of practical geometry was envisaged (PROGRAMMA DAS ESCOLAS PUBLICAS, 1906, p. 1).

FINAL CONSIDERATIONS

Whether they begin with definitions or preliminary notions, the 19th century books of theoretical or practical geometry mentioned here cannot escape this practice that began with Euclid's Elements, namely, to introduce the basic concepts into the book's first pages of the book.

Vasco de Araujo e Silva introduced in his book basic concepts such as the straight, when the students themselves draw freehand representations of them, however, he did not escape the tradition of listing in these initial pages more or less formalized definitions, as to legitimize his text to follow a pattern from Euclidean geometry textbooks. Practical geometry was used by the author in the sense of a knowledge that involves the construction of drawings and measurement.

In proposing to the teacher that to exercise the pupils in the first elements of geometry, to familiarize them with these figures, to enable them to use the geometrical instruments, whether to check if the representation made on the slate is correct or to trace them, correctly, it implies that the student must be accustomed to this kind of thinking, which differs from arithmetic thinking.

Vasco de Araujo e Silva suggests that to learn geometry the student needs to visualize; to exercise; get used to the use of geometric instruments; build cardboard models to visualize geometric solids; gradually know the geometric ones; begin the study by Plane Geometry, then move to Space Geometry. In addition, he proposes the comparison exercise, using an appendix where the flat geometric figures are used as reference to check the representations proposed in the exercises.

The book dialogues with the teacher and the student, but the responsibility for leading the process is the teacher, who suggests the student to do, observe, trace, describe, form, measure, find, evaluate and draw. The proposed exercises do not include questionnaires with questions such as Borges (1882), but basically construction tasks. Being a textbook for primary education, it avoided very formal definitions, escaped other definitions that perhaps the student, at this level of education, could not understand how (dimension, concave, convex, angle measure, scale, degree, among others), presented no demonstrations and theorized little.

Returning to the investigative question, we conclude that, for this author, geometric knowledge for primary education must be practical, elementary; teaching should start with freehand drawing, handling of geometric_instruments and use of carton. Learning geometry implies, therefore, in visualizing, drawing, experimenting, constructing geometric objects through step-by-step instructions, exercising gradually and without much theorizing. Despite that, it is possible to see the presence of definitions' trails in the style of Euclid's Elements in this author's text.

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